



Effect of intermittent edge support on in-plane plate compressive strength

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Abstract

The objective of this paper is to evaluate the in-plane response of steel plates when edge support conditions are intermittent on the unloaded edge, instead of continuous. The study is motivated by the expected boundary conditions for the top plate of a developed prototype all-steel modular floor assembly. Current AISC 360 provisions do not provide explicit guidance for local buckling in these conditions. A material and geometric nonlinear shell finite element modeling protocol is developed and validated for steel plates in ABAQUS. The protocol is applied in a parametric study in which support spacing and plate slenderness are systematically varied. Potential design methods that consider limit states of (i) buckling between supports and (ii) plate buckling are explored. A simplified formula for the minimum required connector spacing to develop the plate strength is provided. The results are intended to inform hand calculations of the floor assembly and are part of a larger project investigating the strength, vibration and acoustical performance of the assembly.

1. Introduction

Stiffness and strength of plates loaded in their plane are highly sensitive to loading and edge boundary conditions. Classical plate stability solutions show that an end-loaded plate with simple continuous supports on all four sides buckles at a stress 9.4 times higher than the same plate with one unloaded edge unsupported. In built-up members it is common to desire the use of intermittent connection between the elements as opposed to continuous connection. However, AISC 360 (2022) provides little more than commentary guidance on this condition, leaving the engineer to provide continuous connection or ignore the beneficial composite behavior. Recent work on a new all-steel modular floor assembly has re-opened interest in the performance of large steel plates with intermittently connected supports. This paper develops and discusses a shell finite element study to address this condition. The paper first provides details on the finite element modeling of plates to establish appropriate element selection, element discretization, and imperfection magnitudes. This is followed by discussion of potential design treatments for the condition. The parametric study results on the intermittently supported plates are compared with the simplified design treatments with alternatives discussed and recommendations made.

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2. Plate Finite Element Modeling

Prior to performing parametric analysis of compressed plates with the unloaded edges intermittently supported classical plate studies were performed in ABAQUS to verify the modeling protocol. These studies are provided here as a validation step for the later parametric analysis.

2.1 Element Selection and Discretization

Motivated by the floor plate in the prototype FastFloor modules a rectangular plate with length, $a=180$ in. (4572 mm), width, $b=60$ in.(1524 mm), and thickness, $t=0.375$ in. (9.525 mm) is modeled in ABAQUS. The plate is modeled with isotropic material $E=29000$ ksi (199955 MPa), $\nu=0.3$ and simply supported on all edges. A reference compressive load of 1 kip/in. (177 N/mm) is applied along the transverse edges as shown in Fig. 1.

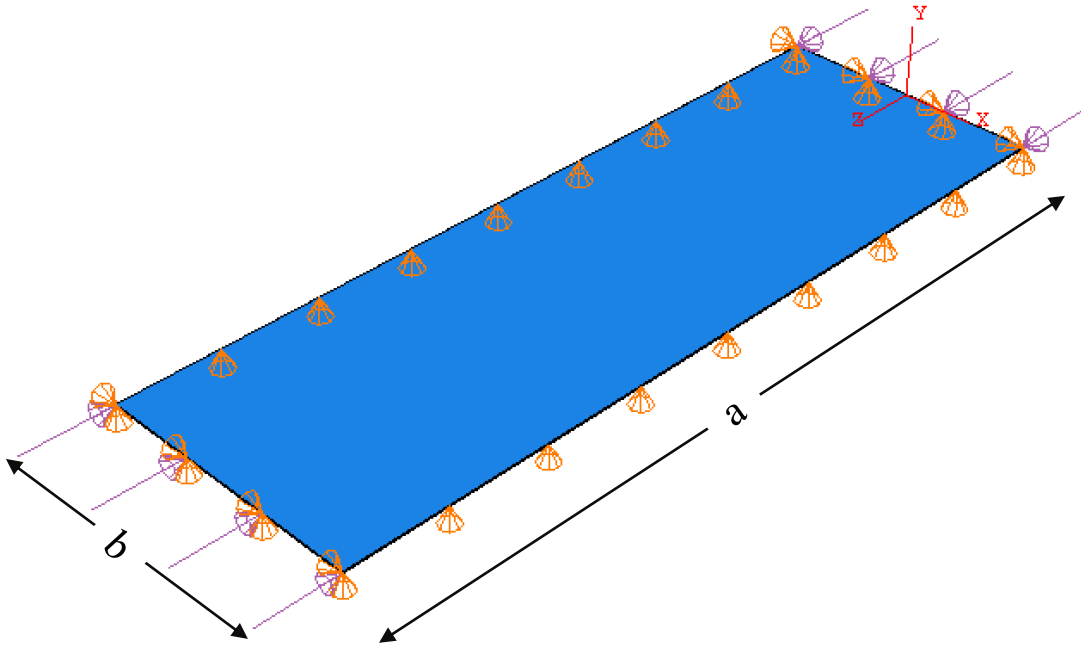


Figure 1. Loading and boundary conditions of studied plate in ABAQUS

ABAQUS provides several shell elements potentially appropriate for this study: a) S4R – a 4-noded thin plate quadrilateral element that uses linear interpolation and can be used to capture large strains in shells; S4R uses a reduced integration scheme to lower the computational cost and remove parasitic shear modes; b) S4 – a general purpose fully integrated 4-noded quadrilateral shell element, c) S8R – a curved, thin shell 8-noded element that uses quadratic interpolation and reduced integration, and d) S9R5 – a curved thin shell 9-noded quadrilateral element that includes the internal drilling degree of freedom in addition to the boundary nodes.

If we define the number of elements along the longitudinal direction as n_a and the number of elements along the transverse direction as n_b , then the element length on the loaded edge is b/n_b , and on the unloaded edge a/n_a , while the element aspect ratio is $(a/n_a)/(b/n_b)$. The shell elements, mesh (edge) discretization, and element aspect ratios are varied, and linear bifurcation analysis (LBA) is performed in ABAQUS to investigate the elastic local buckling of the plate. Conclusions are drawn on the appropriate element discretization and shell element based on the convergence to the critical buckling force. The results from ABAQUS are validated by comparing them to the analytical solution for critical plate buckling (e.g., Timoshenko et al., 1989):

$$N_{cr} = \frac{\pi^2 D}{a^2} \left(m + \frac{1}{m} \cdot \frac{a^2}{b^2} \right)^2 \quad (2.1.1)$$

where, $D = Et^3/12(1 - \nu^2)$ is the plate rigidity and m is the number of buckling waves in the longitudinal direction. For our studied plate $N_{cr} = 1.53$ kip/in. (271 N/mm) and $m=3$ for the first mode. Results of the element selection and discretization study are provided in Fig. 2.

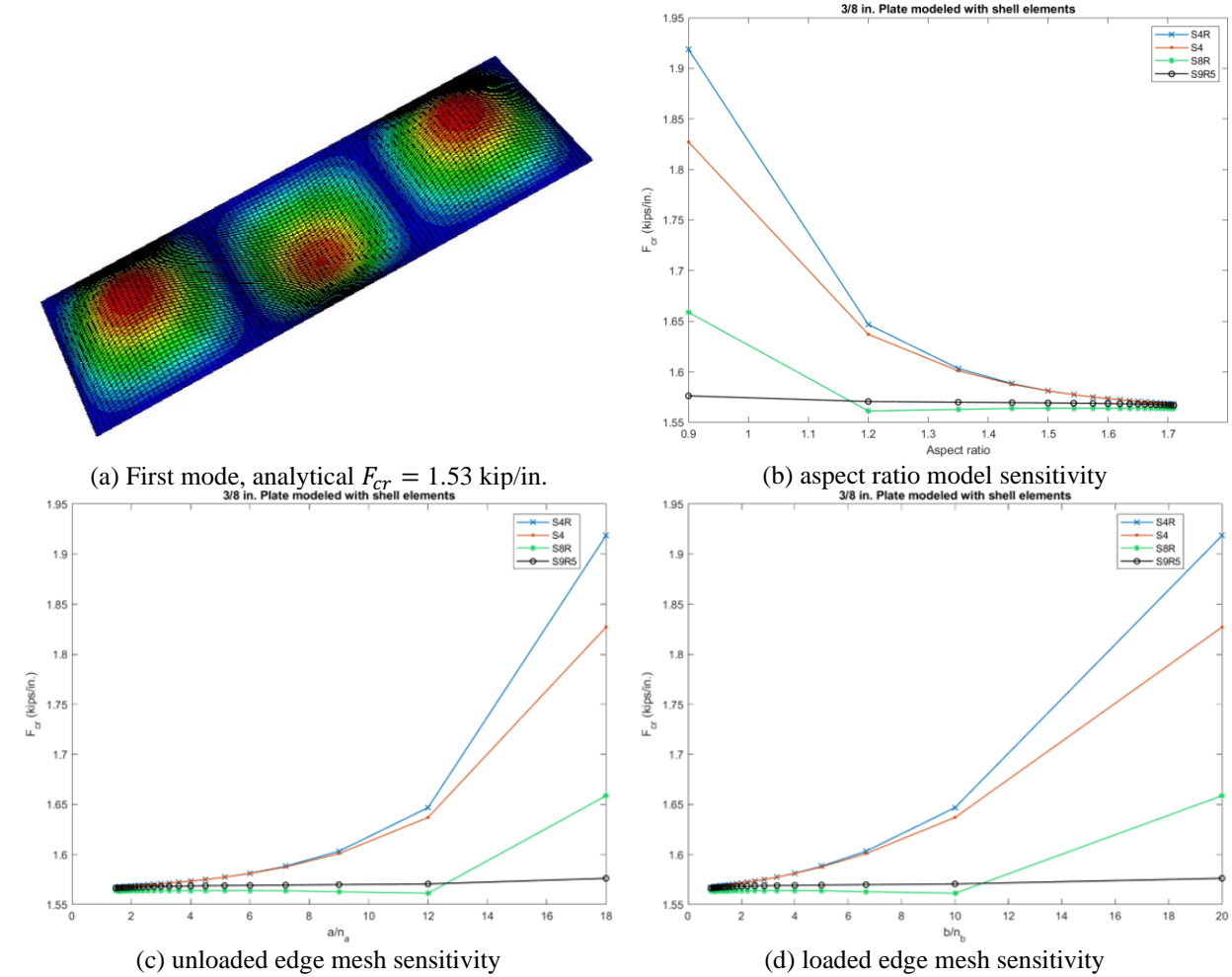


Figure 2. Element selection and discretization study on plate buckling in ABAQUS

The quadratic thin shell elements: S8R and S9R5 significantly outperform the linear elements, even for relatively coarse meshes. Note, S9R5 is preferred; however, it is not available in the ABAQUS CAE graphical user interface and thus engineers only using CAE and not low-level input commands are thus recommended to use S8R. Studies in this work employ the S9R5 element.

For the S4 or S4R element the study shows elements no larger than 4 in. longitudinal and 3 in. transverse are required; while for the S8R or S9R5 element size may be as large as 12 in. longitudinal and 10 in. transverse. Given the buckling half-wavelength is b , or 60 in., this implies as many as 20 elements are required for convergence with the linear elements, but as little as 5 for the quadratic elements.

2.2 Plate Collapse Simulation Validation

As a second exercise prior to the parametric study, we performed shell finite element collapse simulations of simply supported plates loaded on their transverse edge. These simulations require a geometric and material nonlinear analyses with imperfections (known as a GMNIA model). The material is assumed elastic-plastic with $F_y = 50$ ksi (345 MPa). Geometric imperfections are assumed to be in the shape of the first mode, with three different magnitudes considered: $0.1t$, $0.2t(F_y/F_{cr})$, or $0.2t\sqrt{F_y/F_{cr}}$. (See ABAQUS (2024), Dawson and Walker (1972), Farzanian et al. (2018) for more on imperfection modeling.) To achieve variation in the plate slenderness we considered models that varied b with a fixed F_y , or varied F_y with a fixed b – thickness, t , was held constant at 0.375 in. (9.525 mm). Results of the study are provided in Fig. 3, where the simulation results are compared to Winter’s effective width expression, i.e.:

$$\rho = \begin{cases} 1 & \lambda \leq 0.673 \\ (1 - 0.22/\lambda)/\lambda & \lambda > 0.673 \end{cases} \quad (2.2.1)$$

where

$$\lambda = \sqrt{F_y/F_{cr}}, \quad F_{cr} = k \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{b}\right)^2, \quad k = 4$$

Note, the prototype FastFloor plate has a slenderness of 2.6 and is highlighted in Fig. 3. The results show that the ABAQUS collapse simulations with an appropriate element, mesh, boundary conditions, material model, and imperfections provide predictions well aligned with the empirically derived effective width expression (or vice-a-versa the Winter expression is a good fit to the actual mechanics). The model is not overly sensitive to how the variation in slenderness is produced, nor to the choice of imperfection magnitude (note t is constant in the studied models).

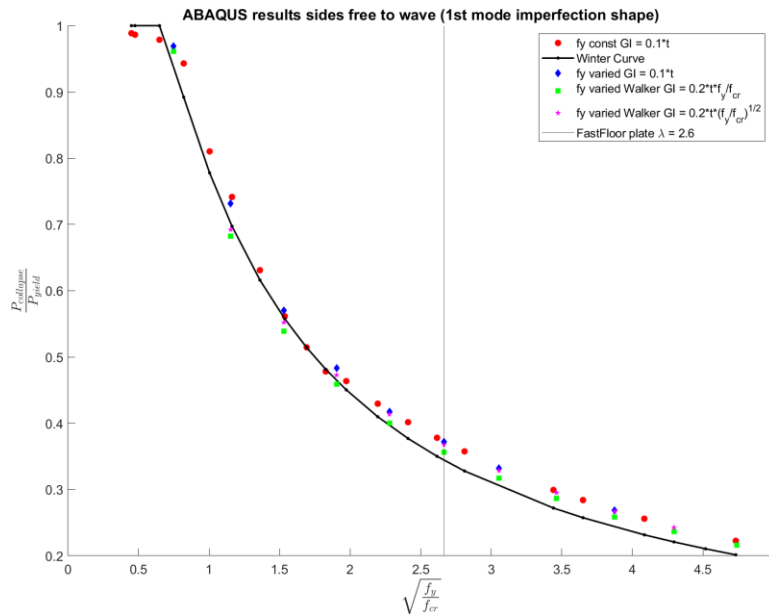


Figure 3: GMNIA plate strength study, compared with Winter’s strength curve

2.3 Compressed Plates with Intermittent Supports on the Unloaded Side

A series of ABAQUS models are created to investigate the in-plane plate compressive strength when intermittent support conditions are provided on the unloaded side, as shown in Fig. 4.

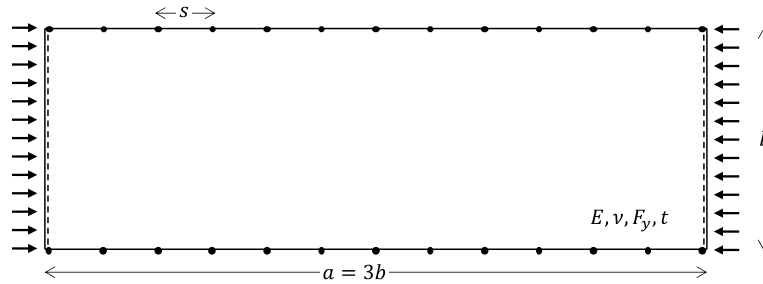


Figure 4. Basic setup and variables for studied plate

In the study, plate widths considered are varied from $b=5$ in. (127 mm) to $b=60$ in. (1524 mm) and the length of the plate a is set to be equal to $3b$ and the plate thickness t at 0.375 in. (9.525 mm). Given a constant F_y of 50 ksi (345 MPa) this results in non-dimensional plate slenderness λ varying from 0.3 to 3.5. The spacing s of the discrete supports along the longitudinal (unloaded) edges of the plates are varied from 0.5 in. (12.7 mm) to b in., thus providing a wide range of s/b ratios with a maximum s/b of 1. LBA analysis is performed based on the protocols established in Sec. 2.1 and are utilized as the imperfection shape. GMNIA collapse analyses are performed as described in Sec. 2.2 with imperfection magnitude set at $0.1t$. The intermittent, discrete, simple supports are modeled as perfect, i.e. no fastener flexibility is included. The plates are modeled until collapse under a displacement-based loading with full results provided in Section 4.

3. Empirical/design plate strength predictions

Here we consider two simplified empirical models for predicting the in-plane compressive strength of a plate with the long sides intermittently supported. First, a simple limit state model where we treat buckling between the fasteners as a simply supported column and use AISC’s column curve (AISC 360 (2022)) and compare that strength with the plate buckling strength as predicted by Winter’s effective width expression (in the Peköz (1986) format). Second, an interacted model that follows Peköz’s (1986) unified approach in concept – utilizing the column prediction between supports as a limiting stress and then calculating the plate strength with the effective width expression limited to this column stress. Consistent with Fig. 4 our problem is: what is the predicted strength of an end loaded plate of length, a , width b , thickness t , continuously simply supported on the loaded edges but intermittently provided simple supports on the unloaded edge, at spacing s , with material modulus, E , Poisson’s ratio ν , and yield stress F_y ?

3.1 Limit State Model

Assume the plate strength is defined as

$$P_{nLS} = \min(P_s, P_w) \quad (1)$$

where, P_s is the strength of the plate assuming global “column” buckling between fasteners, i.e

$$P_s = \begin{cases} 0.658\lambda_s^2 & \lambda_s \leq 1.5 \\ \frac{0.877}{\lambda_s^2} & \lambda_s > 1.5 \end{cases} \quad (2)$$

where, $\lambda_s = \sqrt{\frac{P_y}{P_{crs}}}$, $P_y = btF_y$, and $P_{crs} = \frac{\pi^2 EI}{s^2} = \frac{\pi^2 Ebt^3}{12s^2}$.

Note, to compare across different plates we may consider the normalized capacity:

$$\rho_s = P_s/P_y \quad (3)$$

Further, P_w is the strength of the plate assuming Winter's effective width is valid, i.e.

$$P_w = \begin{cases} P_y & \lambda_w \leq 0.673 \\ P_y(1 - 0.22/\lambda_w)/\lambda_w & \lambda_w > 0.673 \end{cases} \quad (4)$$

where, $\lambda_w = \sqrt{\frac{P_y}{P_{crw}}}$, $P_y = btF_y$, and $P_{crw} = btf_{crw} = btk \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{b}\right)^2$.

For the plate under study $k = 4$ is appropriate. The normalized capacity, when needed is

$$\rho_w = P_w/P_y \quad (5)$$

3.2 Interacted Model

In the interacted model the local plate slenderness is modified by the global column strength, namely the strength is defined as:

$$P_{nl} = \begin{cases} P_s & \lambda_{ws} \leq 0.673 \\ P_s(1 - 0.22/\lambda_w)/\lambda_w & \lambda_{ws} > 0.673 \end{cases} \quad (6)$$

where, $\lambda_{ws} = \sqrt{\frac{P_s}{P_{crw}}}$, P_s and P_{crw} as previously defined.

3.3 Predicted maximum spacing

To provide insight on when P_s or P_w controls in the limit state model contours of the ratio of P_s/P_w were created and provided in Fig. 5. Notably, the contour $P_s/P_w = 1$ is of interest, as it provides a means to ascertain the limiting spacing, s , at which plate buckling is assumed to control and the intermittent fastener spacing no longer influences the solution.

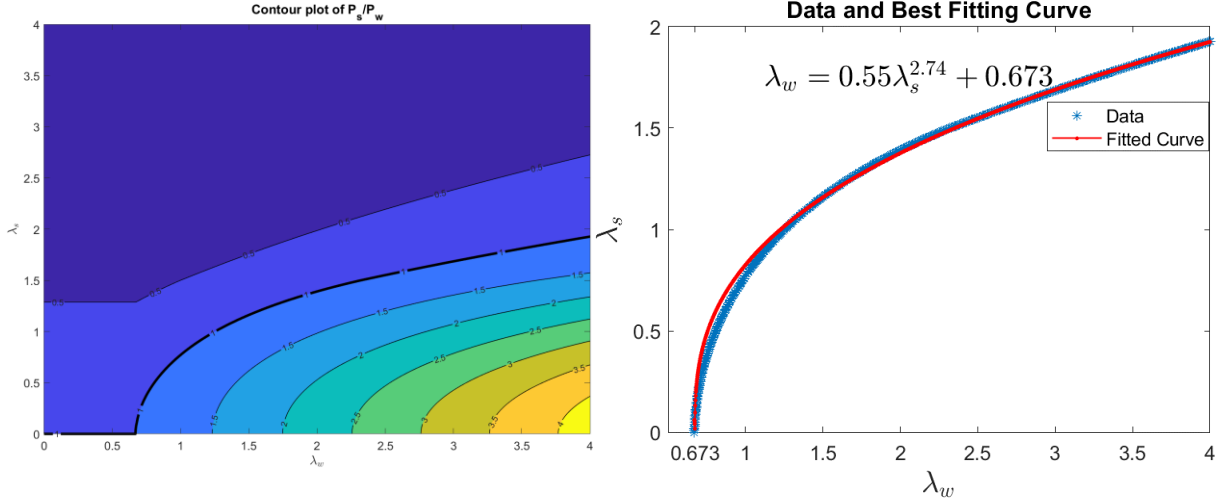


Figure 5. Ratio of predicted buckling strength between fasteners P_s to plate buckling strength P_w
(a) contour plot and (b) fitted curve to $P_s = P_w$

From Fig. 5b, $P_w = P_s$ when

$$\lambda_w = 0.55\lambda_s^{2.74} + 0.673 \quad (7)$$

Solving for λ_s for plate buckling to control (and not buckling between fasteners):

$$\lambda_s \leq \left(\frac{\lambda_w - 0.673}{0.55} \right)^{1/2.74} > 0 \quad (8)$$

Simplifying λ_s by substituting P_y and P_{crs} :

$$\lambda_s = \frac{s\sqrt{12}}{\pi t} \sqrt{\frac{F_y}{E}} \quad (9)$$

which finally results in the following criteria for the maximum spacing:

$$s \leq \frac{\pi t}{\sqrt{12}} \left(\frac{\lambda_w - 0.673}{0.55} \right)^{1/2.74} \sqrt{\frac{E}{F_y}} > 0 \quad (10)$$

Or in an alternative non-dimensional form:

$$\frac{s}{b} \leq \frac{\pi}{\sqrt{12}} \frac{t}{b} \left(\frac{\lambda_w - 0.673}{0.55} \right)^{1/2.74} \sqrt{\frac{E}{F_y}} > 0 \quad (11)$$

Note, a fully explicit expression is possible by expanding λ_w , but to calculate the plate strength λ_w must be known, so the advantage of the expanded expression seems relatively minor.

4. Shell Finite Element Results and Comparison to Strength Models

The shell finite element models exhibit two distinct buckling modes, as shown in Fig. 6 for the first eigen-buckling mode shape of a steel plate with $b/t = 30$ and fasteners spaced at s/b of $1/5$ and $5/6$.

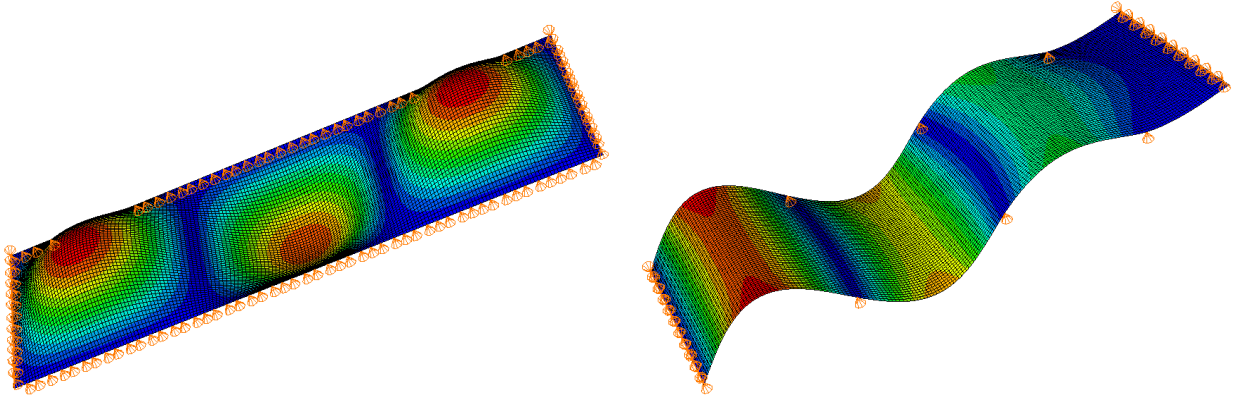


Figure 6. Local buckling and global buckling modes shown for $b=30''$ plate with fastener spacing $5''$ and $25''$

Collapse analysis is conducted in ABAQUS as detailed in Sec. 2.3. The resulting strengths are normalized by the squash load and compared with the empirical strength models in Fig. 7 and 8. In addition, the numerical statistics for the predicted strength from ABAQUS (P_{FE}) divided by the predicted strength from the empirical methods (P_{nLS} or P_{nl}) are provided in Table 2.

Table 2: Statistical comparison of empirical strength models with simulations

b/t	λ_w	P_{FE}/P_{nLS}		P_{FE}/P_{nl}	
		mean	CoV	mean	CoV
13.3	0.29	0.953	0.071	0.953	0.071
26.7	0.58	0.960	0.083	0.960	0.083
40.0	0.87	1.020	0.064	1.154	0.069
53.3	1.16	1.047	0.072	1.390	0.154
66.7	1.46	1.096	0.143	1.527	0.226
80.0	1.75	1.093	0.131	1.359	0.213
120.0	2.62	1.248	0.197	1.383	0.188
160.0	3.49	1.088	0.162	1.394	0.186
All data		1.063	0.115	1.281	0.149
$P_s \leq 0.8P_w$		1.208	0.175	1.230	0.206
$0.8P_w < P_s \leq 1.2P_w$		0.953	0.105	1.137	0.209
$P_s > 1.2P_w$		1.012	0.067	1.405	0.198

$E = 29000$ ksi, $F_y = 50$ ksi, $t = 0.375$ in. [$E = 199955$ MPa, $F_y = 345$ MPa, $t = 9.525$ MPa.]

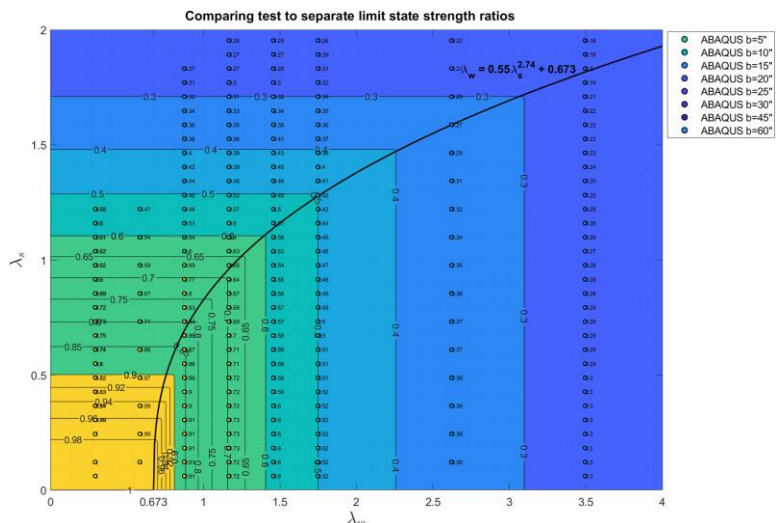
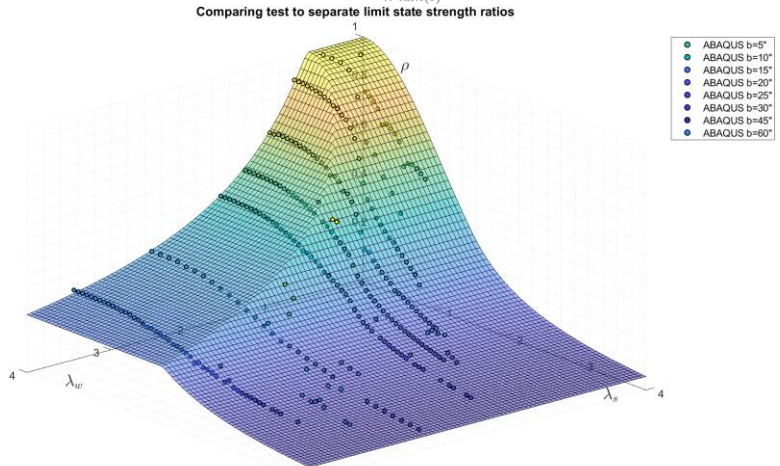
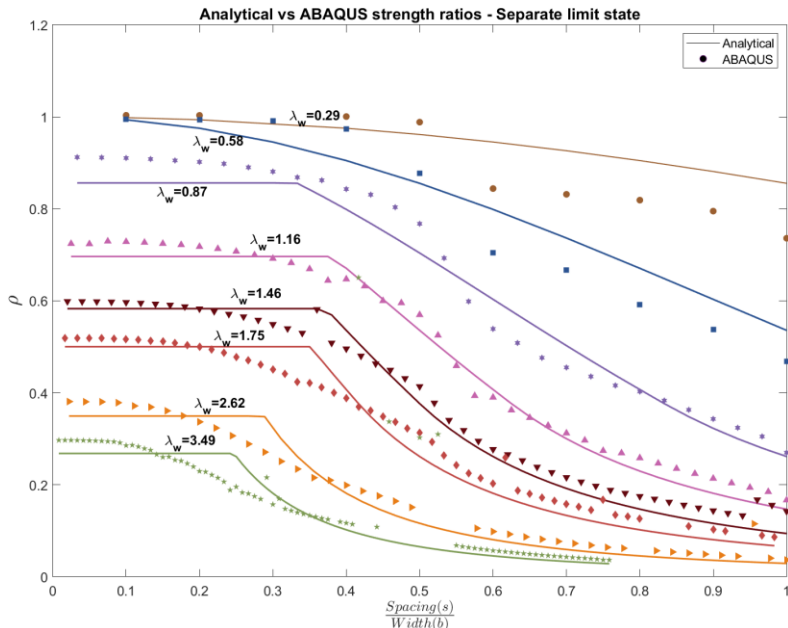


Figure 7. Comparison of normalized strength from ABAQUS with Empirical Limit State Model

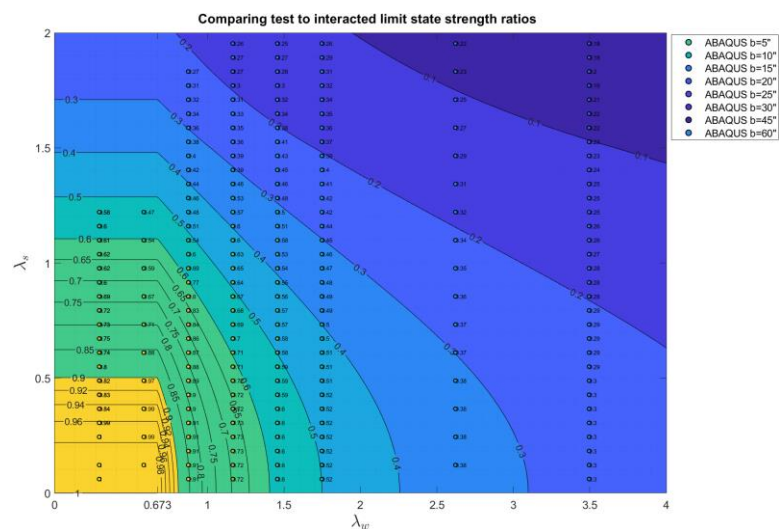
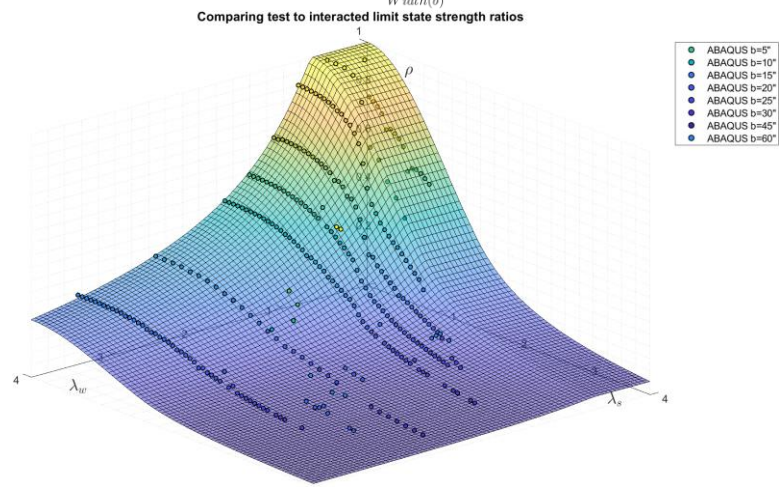
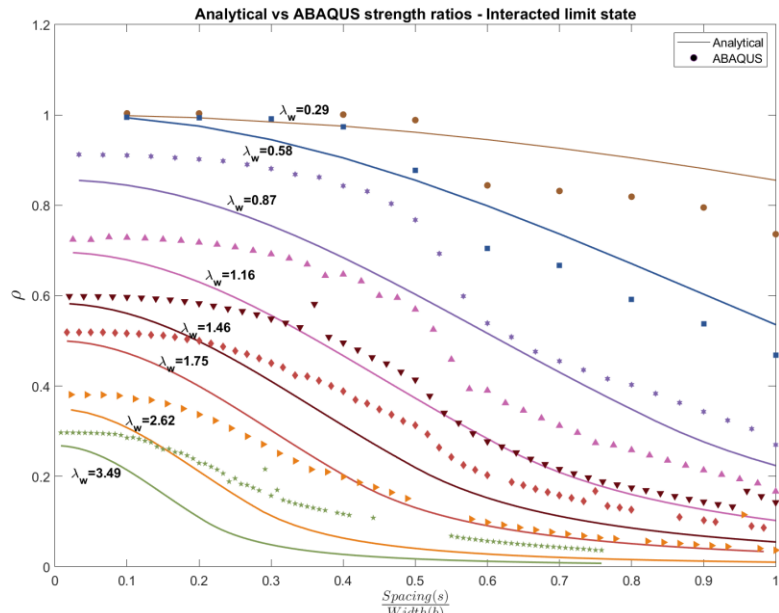


Figure 8. Comparison of normalized strength from ABAQUS with Empirical Interaction Model

For the separate limit state model (P_{nLS} , Fig.7) the design expressions provide a prediction of the s/b at which plate buckling will control the strength. In Table 3 we compare this prediction to the strength level (and corresponding s/b) at which the collapse simulations begin predicting capacities lower than the Winter plate buckling strength.

Table 3: Comparison of predicted limiting s/b with simulations

b/t	λ_w	s/b limit			
		LS model	FE	Simplified ^b	FE/simplified
13.3	0.29	0 ^a	0.5	0.33	1.52
26.7	0.58	0 ^a	0.5	0.33	1.52
40.0	0.87	0.38	0.33	0.32	1.03
53.3	1.16	0.39	0.33	0.28	1.19
66.7	1.46	0.37	0.23	0.25	0.93
80.0	1.75	0.35	0.22	0.23	0.97
120.0	2.62	0.29	0.19	0.19	1.03
160.0	3.49	0.25	0.16	0.16	1.00

$E = 29000$ ksi, $F_y = 50$ ksi, $t = 0.375$ in. [$E = 199955$ MPa, $F_y = 345$ MPa, $t = 9.525$ MPa.]

a) LS model predicts $s/b=0$ required if $\lambda_w \leq 0.673$ (fully effective)

b) simplified model as provided in the discussion

4. Discussion

The shell finite element models demonstrate that intermittently connected plates can develop the in-plane compressive plate strength of a continuously connected plate. For the studied plates the required spacing, s/b , from the FE simulations varies from 0.16 to 0.50 with the most locally slender plates requiring the tightest spacing. The limit state model does not accurately capture the transition, so an alternative is sought. The limiting s/b is empirically derived as a function of λ_w :

$$\frac{s}{b} \leq \frac{0.3}{\sqrt{\lambda_w}} < 0.33 \quad (4.1)$$

An upperbound is placed on s/b to insure some limited redundancy in the provided number of support points. Accuracy of this simple expression is provided in Table 3, and the result is satisfactory and therefore recommended for use in practice. It is important to note that in this study fastener flexibility is not considered, as such, it is presumed that the local stiffness at the fasteners is high enough to be sufficiently ignored – if that is not the case the s/b limits will need to be further decreased in an intermittently connected plate. In addition, this work examines only point supports, a stitch weld, or other intermediate support with a finite length is not considered, and could lead to more relaxed spacing requirements depending on support length.

For the general case where the engineer wants to predict the strength independent of the fastener spacing, two models were considered: the separate limit state model (P_{nLS}) and the interacted model (P_{nI}). The interacted model is easy to implement, follows the logic of local-global interaction that is utilized in design specifications today, but is too conservative to recommend. Interaction of the type assumed in the model is not observed in the simulations. The limit state model provides reliable predictions at its two limits, but over-estimates the strength in the region

in which the failure is transitioning between plate buckling and buckling between the fasteners (Note the results for $0.8P_w < P_s \leq 1.2P_w$ in Table 2). Additional modifications are needed to provide a robust solution, though as a method for preliminary strength prediction it provides all necessary features.

The authors are also interested in the in-plane shear strength and stiffness of intermittently connected plates and will extend their current study to those conditions in the near future.

5. Conclusions

Shell finite element simulations of compressed plates with intermittently connected simple supports on their unloaded side were completed. The simulations covered a broad range of plate slenderness and support spacing. It was found that intermittently supported plates can develop their full plate compressive capacity. However, the required spacing decreases as the plate becomes more slender. In general, fastener spacing divided by plate width in the ratio of 0.15 to 0.30 is found to be necessary for developing full plate compressive capacity. Expressions are developed and provided for the required spacing. Work is underway to extend the studies to in-plane shear stiffness and strength of intermittently supported plates.

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References

- ABAQUS (2016). “ABAQUS Analysis Users Manual.” Accessed: Feb. 26, 2024. [Online]. Available: <http://130.149.89.49:2080/v2016/books/key/default.htm>
- AISC 360 (2022) “ANSI/AISC 360-22 Specification for Structural Steel Buildings” American Institute of Steel Construction, Chicago, IL.
- Dawson, R. G. and Walker, A. C. (1972). “Post-buckling of geometrically imperfect plates.” American Society of Civil Engineers Journal of the Structural Division, 98(1), 75-94.
- Farzarian, S., Louhghalam, A., Schafer, B.W., Tootkaboni, M. (2018). “Geometric Imperfections in Shell Finite Element Models of CFS Members - A Review of Current State of Practice.” Proceedings of the Annual Stability Conference Structural Stability Research Council Baltimore, Maryland, April 10-13, 2018.
- Peköz, T. (1986). “Development of a Unified Approach to the Design of Cold-Formed Steel Members.” Report to the American Iron and Steel Institute, Washington, D.C..
- Timoshenko, S.P., and Gere, J.M (1961) Theory of Elastic Stability, Unabridged 2nd Ed., Dover (2009).