



Critical moment of doubly-symmetric beams with prebuckling deflection: the effect of end supports

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Abstract

In the reported research the effect of prebuckling in-plane deflections on the critical moment to lateral-torsional buckling of beams is investigated. In this paper there is a special focus on the influence of end support conditions. Analytical derivations are completed for selected end supports, which lead to closed-form equations to calculate the critical moment with considering the prebuckling deflection. Numerical studies are performed, too: critical moment values were calculated with and without considering the effect of prebuckling deflection, using the derived analytical formula, beam finite element method, and shell finite element method. In general, the results from the various methods show reasonable coincidence, though some discrepancies are also revealed. One of the most important conclusions is that – unlike suggested by the literature, – the prebuckling deflections are not always positive, but for certain end supports can decrease the critical moment.

1. Introduction

Lateral-torsional buckling (LTB) is a classic type of instability of beams. A classic and simple analysis approach is the so-called *linear buckling analysis* (LBA), which assumes a perfectly elastic and imperfection-free structure, but considers some nonlinear terms in the geometric equations. LBA leads to the buckled shapes and corresponding critical load factors. In the case of LTB, the critical load is typically represented by the *critical moment*. Though stability phenomena are influenced by the imperfections, still, LBA is a prominent tool to understand buckling, and widely employed in many design procedures.

In classic LBA, the equations to be solved are written on the original, undeformed structure. It was observed, however, that the primary, first-order deformations might influence the solution. As the load is increased on the structure, the primary deflections increase, and by the time when buckling occurs, the structure is already deflected. If this deflected shape (which is independent of the imperfections, since it exists even if the original structure is perfect) is considered in the LBA, the associated critical load is different from the one without considering the prebuckling deflection. It is reasonable to assume that prebuckling deflection is never zero, however, whether it has important or negligible effect on the buckling, is dependent on the structure.

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In the case of beams subjected to LTB, the effect of the prebuckling deflection was included even in the very first analytical solution for the LTB problem, in Michell (1899). Later, the solution without the prebuckling deflection effect became widely known from the work of Timoshenko (1910), who clarified the importance of warping, and published the well-known classic formula for the critical moment for LTB. The influence of prebuckling displacement is discussed in a relatively small number of papers, e.g., in Trahair and Woolcock(1973),Vacharajittiphan et al. (1974), Roberts and Azizia (1983), Pi and Trahair (1992), Attard and Kim (2010), Erkmén and Attard (2011).

There seems to be a consensus in the available literature that the prebuckling deformations increase the critical moment, and that the increment is dominantly determined by the lateral rigidity of the beam. However, there are some discrepancies, too, both in the proposed analytical expressions and in the numerical results. Moreover, the reported researches mostly focused on the simplest case of LTB, namely: simply supported single-span beams, subjected to uniform bending, and with doubly symmetric I-shaped cross-sections. Other cases are hardly investigated. To better understand the role of prebuckling deflections on LTB, we started a research project.

This paper focuses on the end supports. In Section 2 an analytical derivation is shown which is applied to selected end supports in Section 3, leading to closed-form equations to calculate the critical moment with considering the prebuckling deflection. Numerical studies are presented in Section 4: critical moment values calculated with and without considering the effect of prebuckling deflection are calculated, using the derived analytical formula, beam finite element method, and shell finite element method. In general, the results from the various methods show reasonable coincidence, though some discrepancies are also revealed, as summarized in Section 5.

2. Analytical derivation

2.1 General

In this paper the derivation is employed as in Pi and Trahair (1992). First the derivation steps are briefly summarized, then the derivation is applied for beams with various support conditions. In all the discussed cases the beam is assumed to have doubly symmetric cross-section, and the beam is subjected to uniform moment. The problem is illustrated in Fig. 1, (for forked supports and I-shaped cross-section,) where the considered coordinate axes and the u , v , and w translations are shown, too.

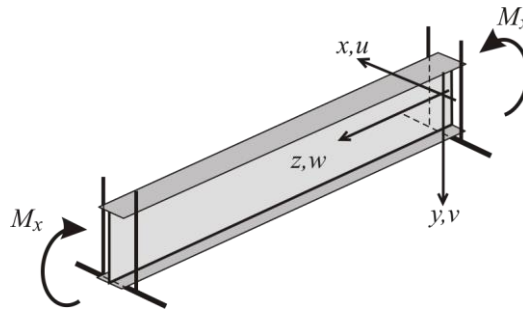


Figure 1: The discussed beam problem, and basic notations.

2.2 Total potential

The energy method is applied. The total potential is composed of the strain energy and the work as follows:

$$\Pi = S - W \quad (1)$$

where

$$W = \int_0^L \int_A \varepsilon_z \frac{M_x}{I_x} y dA dz \quad (2)$$

$$S = \frac{1}{2} \int_0^L \int_A EI_y \kappa_y^2 + EI_w \kappa_{zd}^2 + GJ \kappa_z^2 dA dz \quad (3)$$

where M_x is the applied (uniform) moment, I_x and I_y are the second moments of area for the x and y axes, respectively, I_w is the warping modulus, J is the torsional inertia, ε_z is the (second-order) longitudinal normal strain, κ_y is curvatures of the beam axis (in the lateral direction), κ_z is the rate of change of the twisting rotation, κ_{zd} is the derivative of κ_{zd} with respect to z , and E and G are the Young's modulus and shear modulus, respectively.

2.3 Curvatures

In the calculation of the κ curvatures some higher-order terms must be considered. The considerations are not detailed here, but the same formulae are used as in Pi and Trahair (1992), as follows:

$$\kappa_y = u'' + \varphi v'' \quad \kappa_z = \varphi' - \frac{1}{2} u' v'' + \frac{1}{2} u'' v' \quad (4)$$

2.4 Longitudinal normal strain

To calculate the work of the loads/stresses, we need the strains. Now we have longitudinal stresses only, hence the only strain term needed is the longitudinal normal strain. This is derived from the u , v , and w translations, based on the Green-Lagrange strain tensor, as follows:

$$\varepsilon_z = \frac{\partial w}{\partial z} + \frac{1}{2} \left(\left(\frac{\partial u}{\partial z} \right)^2 + \left(\frac{\partial v}{\partial z} \right)^2 \right) \quad (5)$$

Since the strains should be calculated on the deformed geometry, the above expression leads to a rather lengthy formula. If we keep the second-order term and one single third-order term, the (nonlinear) longitudinal normal strain over the cross-section is approximated as follows:

$$\varepsilon_z = \left(\varphi u'' + \frac{1}{2} \varphi^2 v'' \right) y \quad (6)$$

2.5 Displacement functions

In order to be able to calculate the total potential, we need displacement functions. For the primary displacement, we can employ classic strength of materials equations. The moment is uniform along the length, and the supports are pinned in the vertical plane, thus, the displacement of the beam's system line is a quadratic function:

$$v(z) = v_m 4z(L - z)/L^2 \quad (7)$$

where L is the member length, and v_m is the maximum vertical displacement:

$$v_m = \frac{M_x L^2}{8EI_x} \quad (8)$$

The assumed displacement functions for the lateral and torsional displacements are dependent on the supports, and will be discussed in Section 3. However, in each case the displacement functions are expressed by two parameters: the u_m and φ_m displacement amplitudes of the lateral translation and twisting rotation, respectively.

2.6 Critical moment

Considering the above displacement functions, the total potential can be calculated. The strain energy function will have 11 terms, each of them quadratic expressions of the displacement parameters. Similarly, substituting the normal strain expression into the work formula and performing the integration, the work is obtained. It will have two terms, each of them quadratic expressions of the displacement parameters.

Then, we need to take the partial derivatives of the potential, which should be zero. This leads to a system of two equations. The equations are linear in u_m and φ_m , and the equations are homogeneous. Nontrivial solution exists if the ‘ \mathbf{C} ’ coefficient matrix is singular. A straightforward solution is to take the determinant of the coefficient matrix, and make it equal to zero.

$$|\mathbf{C}| = 0$$

In classic LTB derivations the resulting equation is quadratic in terms of M_x . However, in our case this equation is 4th-degree due to the considered higher-order terms in various strain and curvature expressions. To simplify the problem, we need to apply two approximations. (i) First, we need to eliminate the terms with M_x^4 . Since there are no terms with M_x^3 , now the equation becomes quadratic. (ii) Secondly, we should neglect certain terms. The consideration is that in typical doubly-symmetric thin-walled sections the various flexural and torsional stiffness terms are strongly different. It is assumed, therefore, that:

$$\left(\frac{EI_w/L^2}{EI_x}\right)^2 \cong 0 \quad \left(\frac{GJ}{EI_x}\right)^2 \cong 0 \quad \frac{GJ(EI_w/L^2)}{(EI_x)^2} \cong 0 \quad (9)$$

With these approximations the quadratic equation is solvable and we can have closed-form solution for the critical moment

3. Analytical formulae

3.1 Considered cases

In the plane of the bending both ends are simply supported, i.e., they can freely rotate around the x -axis. From the aspect of LTB, however, it is also important how they are supported against y -axis rotation and warping. Various combinations of rotational and warping supports are considered here. If the end cross-section is free to rotate in the lateral direction (i.e., around the y axis), it will be notated as ‘Pr’, while if it is fixed against lateral rotation, it is notated as ‘Fr’. Similarly, if warping can freely occur, it is ‘Pw’, while if warping is prevented, it is ‘Fw’. Accordingly, for example, PrPw-PrPw identifies the basic case of lateral-torsional buckling, when forked supports

are applied. Or, FrFw-FrFw is the case when both supports are fixed from the viewpoint of LTB (though pinned in the primary direction).

The cross-sections are doubly-symmetric. The most typical such section is doubly-symmetric I-shaped section (abbreviated here as ‘DSI’). However rectangular hollow sections (‘RHS’) are also doubly symmetric and can buckle if the section is relatively narrow. The main difference between DSI and RHS is that an RHS has much larger torsion stiffness compared to a DSI.

3.2 PrPw-PrPw

The assumed displacement functions for the lateral-torsional buckling are simple half sinewaves, in accordance with the assumed forked supports:

$$u(z) = u_m \sin \frac{\pi z}{L} \quad \varphi(z) = \varphi_m \sin \frac{\pi z}{L} \quad (10)$$

where u_m and φ_m are the displacement amplitudes (at the middle of the beam).

By following the derivation steps as summarized above, it is possible a closed-form solution for M_{cr} , which is identical to the one in Pi and Trahair (1992), as follows:

$$M_{cr} = M_{cr0} / \sqrt{\left(1 - \frac{I_y}{I_x}\right) \left(1 - \frac{GJ}{2EI_x} \left(\frac{GJ}{2EI_x} + \frac{\pi^2 EI_w}{2EI_x L^2}\right)\right)} \quad (11)$$

where M_{cr0} is the critical moment without considering the prebuckling deflection:

$$M_{cr0} = \frac{\pi}{L} \sqrt{EI_y \left(GJ + \frac{\pi^2 EI_w}{L^2}\right)} \quad (12)$$

In typical DSI sections:

$$\frac{EI_w/L^2}{EI_x} \cong 0 \quad \frac{GJ}{EI_x} \cong 0 \quad (13)$$

hence the M_{cr} formula can further be simplified to:

$$M_{cr} = M_{cr0} / \sqrt{\left(1 - \frac{I_y}{I_x}\right)} \quad (14)$$

The M_{cr}/M_{cr0} ratio for this case (which will also be called as *reference ratio*), therefore, is:

$$\frac{M_{cr}}{M_{cr0}} = 1 / \sqrt{\left(1 - \frac{I_y}{I_x}\right)} \quad (15)$$

and the relative increment of the critical moment due to prebuckling deflection – which will be referred to here as *reference increment* – is as follows:

$$\frac{M_{cr} - M_{cr0}}{M_{cr0}} = 1 / \sqrt{\left(1 - \frac{I_y}{I_x}\right) - 1} \quad (16)$$

In the case of RHS, it is reasonable to assume that the warping is nearly zero, however, at the same time, GJ is not small anymore. Hence, instead of applying the approximations in Eq (13), we can introduce the following approximations:

$$\frac{EI_w/L^2}{EI_x} \cong 0 \quad \frac{GJ}{EI_x} \gg 0 \quad (17)$$

Accordingly, the simplified formula for the critical moment ratio is:

$$\frac{M_{cr}}{M_{cr0}} = 1 / \sqrt{1 - \frac{I_y}{I_x} - \frac{GJ}{2EI_x} + \frac{GJ}{2EI_x} \frac{I_y}{I_x}} = 1 / \sqrt{\left(1 - \frac{I_y}{I_x}\right) \left(1 - \frac{GJ}{2EI_x}\right)} \quad (18)$$

3.3 FrFw-FrFw

The assumed displacement functions in this case are:

$$u(z) = u_m \frac{1}{2} \left(1 - \cos \frac{2\pi z}{L}\right) \quad \varphi(z) = \varphi_m \frac{1}{2} \left(1 - \cos \frac{2\pi z}{L}\right) \quad (19)$$

where u_m and φ_m are the displacement amplitudes (at the middle of the beam).

The M_{cr0} without prebuckling deflections is:

$$M_{cr0} = \frac{\pi}{0.5L} \sqrt{EI_y \left(GJ + \frac{\pi^2 EI_w}{(0.5L)^2}\right)} \quad (20)$$

For DSI sections the simplified formula is:

$$\frac{M_{cr}}{M_{cr0}} = 1 / \sqrt{1 + \frac{I_y}{I_x} - 2 \left(\frac{I_y}{I_x}\right)^2} = \sqrt{\left(1 - \frac{I_y}{I_x}\right) \left(1 + 2 \frac{I_y}{I_x}\right)} \quad (21)$$

For RHS sections the simplified formula is:

$$\frac{M_{cr}}{M_{cr0}} = 1 / \sqrt{\left(1 - \frac{I_y}{I_x}\right) \left(1 + 2 \frac{I_y}{I_x} - \frac{3GJ}{2EI_x}\right)} \quad (22)$$

3.4 PrPw-FrFw

The assumed displacement functions are as follows:

$$u(z) = u_m \left(\sin \frac{kL z}{L} - \frac{z}{L} \sin(kL)\right) \quad \varphi(z) = \varphi_m \left(\sin \frac{kL z}{L} - \frac{z}{L} \sin(kL)\right) \quad (23)$$

where u_m and φ_m are parameters related to the displacement amplitudes. Moreover, for the kL it must be satisfied that $kL = \tan(kL)$, from which the approximate value of kL is 4.4934.

The M_{cr0} without prebuckling deflections is:

$$M_{cr0} = \frac{kL}{L} \sqrt{EI_y \left(GJ + \frac{(kL)^2 EI_w}{L^2} \right)} = \frac{\pi}{0.6992L} \sqrt{EI_y \left(GJ + \frac{\pi^2 EI_w}{(0.6992L)^2} \right)} \quad (24)$$

For DSI sections the simplified moment ratio formula is:

$$\frac{M_{cr}}{M_{cr0}} = \sqrt{\left(1 - \frac{I_y}{I_x}\right) \left(1 + \frac{2I_y}{3I_x}\right)} \quad (25)$$

For RHS sections the formula is:

$$\frac{M_{cr}}{M_{cr0}} = 1 / \sqrt{\left(1 - \frac{I_y}{I_x}\right) \left(1 + \frac{2I_y}{3I_x} - \frac{GJ}{EI_x} \frac{(kL)^2 + \sqrt{(kL)^2 + 1} - 1}{(kL)^2}\right)} \quad (26)$$

The numerical value of the coefficient of the GJ term is approx. 1.1785.

3.5 PrFw-PrFw

The assumed displacement functions this case are:

$$u(z) = u_m \sin \frac{\pi z}{L} \quad \varphi(z) = \varphi_m \frac{1}{2} \left(1 - \cos \frac{2\pi z}{L}\right) \quad (27)$$

where u_m and φ_m are the displacement amplitudes (at the middle of the beam). It is to be noted that though the above functions satisfy the considered support conditions, they lead to higher critical moment values compared to those obtained as the smallest critical moments from numerical methods.

The M_{cr0} without prebuckling deflections is:

$$M_{cr0} = \frac{\pi}{8/(3\pi)L} \sqrt{EI_y \left(GJ + \frac{\pi^2 EI_w}{(0.5L)^2} \right)} \quad (28)$$

For DSI sections the simplified formula is:

$$\frac{M_{cr}}{M_{cr0}} = 1 / \sqrt{\left(1 - \frac{I_y}{I_x}\right) \left(1 + \frac{I_y}{I_x} \left(\frac{27\pi^2}{256} - 1\right)\right)} \quad (29)$$

Since $27\pi^2/256 - 1 = 0.0409$, this formula is nearly identical to the reference ratio.

For RHS sections the simplified formula is:

$$\frac{M_{cr}}{M_{cr0}} = 1 / \sqrt{\left(1 - \frac{I_y}{I_x}\right) \left(1 + \frac{I_y}{I_x} \left(\frac{27\pi^2}{256} - 1\right) - \frac{GJ}{3EI_x}\right)} \quad (30)$$

3.6 FrPw-FrPw

The assumed displacement functions this case are:

$$u(z) = u_m \frac{1}{2} \left(1 - \cos \frac{2\pi z}{L} \right) \quad \varphi(z) = \varphi_m \sin \frac{\pi z}{L} \quad (31)$$

where u_m and φ_m are the displacement amplitudes (at the middle of the beam). Similarly to the previous case, these displacement functions satisfy the boundary conditions, but approximate.

The M_{cr0} without prebuckling deflections is:

$$M_{cr0} = \frac{\pi}{4/(3\pi)L} \sqrt{EI_y \left(GJ + \frac{\pi^2 EI_w}{L^2} \right)} \quad (32)$$

For DSI sections the simplified formula is:

$$\frac{M_{cr}}{M_{cr0}} = 1 / \sqrt{\left(1 - \frac{I_y}{I_x} \right) \left(1 + \left(\frac{9\pi^2}{16} - 1 \right) \frac{I_y}{I_x} \right)} \quad (33)$$

For RHS sections the simplified formula is:

$$\frac{M_{cr}}{M_{cr0}} = 1 / \sqrt{\left(1 - \frac{I_y}{I_x} \right) \left(1 + \frac{I_y}{I_x} \left(\frac{9\pi^2}{16} - 1 \right) - \frac{3GJ}{EI_x} \right)} \quad (34)$$

4. Numerical studies

4.1 Methodology

Three methods have been employed: (a) analytical formula (as summarized in Section 3), (b) beam finite element, (c) shell finite element.

Classic Linear Buckling Analysis (LBA) is performed on the initial (undeformed) geometry of the structure. This is how any commercial FEM software works, too. In other words, the prebuckling displacements are not considered in an LBA. If we still want to consider them, an iterative procedure is necessary, as follow.

1. First, we perform classic LBA, and calculate M_{cr} (which, in this step, will be equal to M_{cr0}).
2. We perform linear static analysis to get the deflected shape (i.e., the prebuckling shape), using the M_{cr} as load from the previous Step.
3. We use the deflected shape from the previous step, and perform LBA on the deflected shape, from which we obtain a new value for M_{cr} .

Then we repeat steps 2 and 3 till convergence. This procedure is necessary to follow whether the calculation uses beam or shell elements.

When beam finite element calculation is performed, it is important to make sure that the employed beam element properly considers the warping and Saint-Venant torsion of thin-walled members.

In this study we have used the educational Mastan2, by Ziemian et al. (2022), and the commercial Ansys software (Ansys, 2020). Since the results of these two programs are nearly identical for the cases discussed here, only the Mastan2 result will be presented. We have used 16 beam elements along the length, and the load is applied as concentrated moments at the girder ends.

In the case of shell FE calculations, we have used Ansys. The applied finite element is SHELL181, which is a 4-node element with 6 degrees of freedom per node. The out-of-plane behavior of the element is based on the Reissner-Mindlin plate theory, hence the out-of-plane shear deformations are directly considered. This is mechanically different from the analytical solution and from the beam FEM applied, where the shear deformations are disregarded.

For the discretization of the model, the (average) element size was set to 40 mm. The concentrated end moment was put to the model by line pressures acting along the edges of the flange and web elements of the member at the midline of the shell elements.

The supports were defined so that they imitate the beam-like supports as much as possible. The support realization is illustrated in Fig. 2, for both cross-sections. It is to note that the so-called master node is a separate node (not part of the member discretization), though in the case of DSI section its geometric location coincides with one of the nodes of the discretized member.

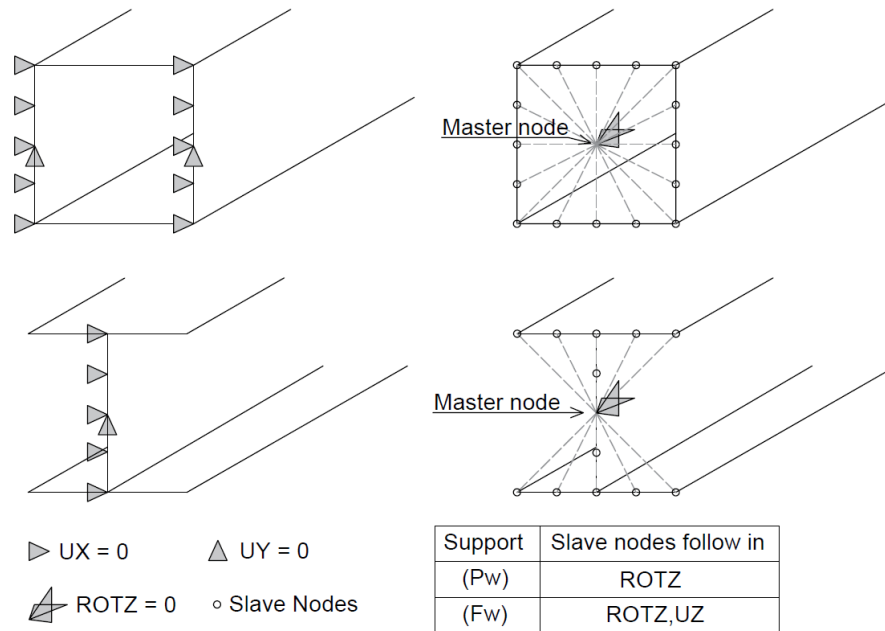


Figure 2: Supports in the shell FEM model.

4.2 Scope

The above-presented analytical formulae have been applied to calculate critical moments with and without considering the prebuckling deflections. The results are compared to those from numerical methods.

A set of DSI sections are considered, defined as follows: the flange width is 200 mm, the flange and web thickness is 20 and 12 mm, respectively, while the total section depth (out-to-out) is a variable taking the following values: 150 mm, 160 mm, 180 mm, 200 mm, 300 mm, 400 mm, and 500 mm.

Similarly, a set of RHS sections are considered, as follows: the section width is 150 mm (out-to-out), the flange and web thickness is 30 and 10 mm, respectively, the total section depth (out-to-out) varies, taking the following values: 150 mm, 160 mm, 180 mm, 200 mm, 300 mm, 400 mm, 500 mm and 1000 mm.

In the case of the simplified analytical formula the member length has no effect on the M_{cr}/M_{cr0} ratio. It is also observed that in the Mastan2 calculations the beam length has marginal effect. Therefore, the presented analytical and Mastan2 results are valid to any beam length. In the case of shell FEM the M_{cr}/M_{cr0} ratios are influenced by the length. This question is not further discussed here, but it is reasonable to assume that the shell FEM results are closer to the beam FEM results if the beam length is large, therefore, long beams have been considered for the shell FEM analyses.

For the material a classic isotropic steel is considered, with a Young's modulus equal to 210000 MPa, and Poisson's ratio equal to 0.3.

4.3 Results

In Figs. 3 to 7 the critical moment increments are plotted in terms of the inertia ratios. It is to mention that in the last figure no shell FEM results are plotted; the reason is that the FrPw support is all but impossible to realize in a shell model.

As far as the DSI results are concerned, all the methods yield similar increments for any support condition. The agreement between the analytical and beam FEM results is particularly good. The shell model usually predicts slightly smaller increments. This is most certainly due to the fact that the shell model is more flexible, allowing various deformations which are excluded from the beam model (e.g., localized deformations, cross-section distortion, shear deformation). The results clearly show the significant influence of the supports. It can be understood that the warping fixity has relatively small influence, but the fixity of the rotation around the minor (vertical) axis has large effect. Moreover, the results suggest that the rotation fixity has a negative effect (i.e., it decreases the moment increment), while the warping fixity can have either a (small) positive or a (small) negative effect.

In the case of the RHS the results are more scattered. When the shell FEM is used, the results are in good agreement with the analytical solution. However, they are affected by the non-beam-like deformations, most notably by the cross-section distortion, due to the fact that a closed cross-section "prefers" to twist with small cross-section distortion which is hard to eliminate. The beam FEM results show large differences compared to the analytical solutions; this is remarkable since in the case of DSI sections the agreement between the analytical and beam model results is excellent.

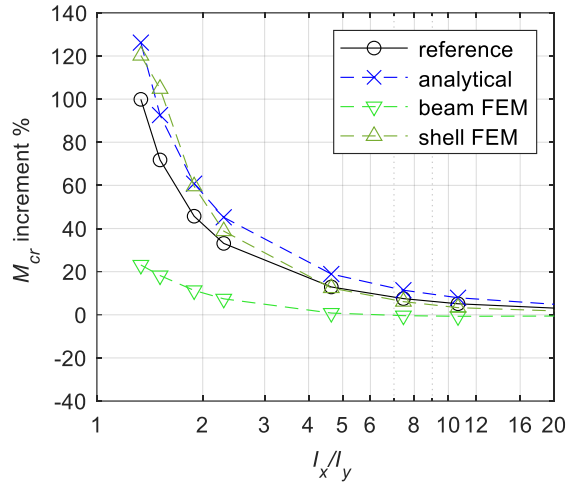
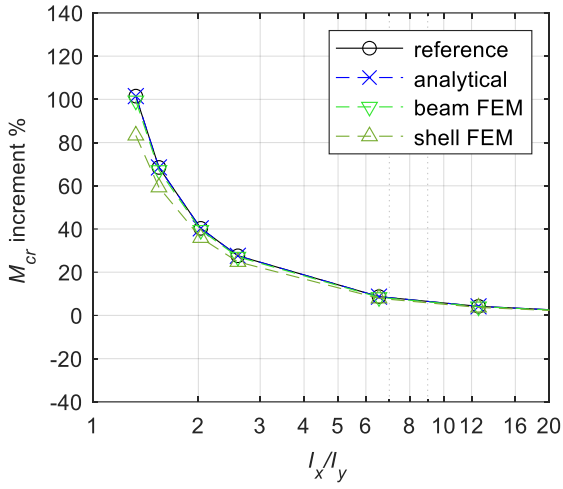


Figure 3: M_{cr} increments, PrPw-PrPw, DSI (left) and RHS (right)

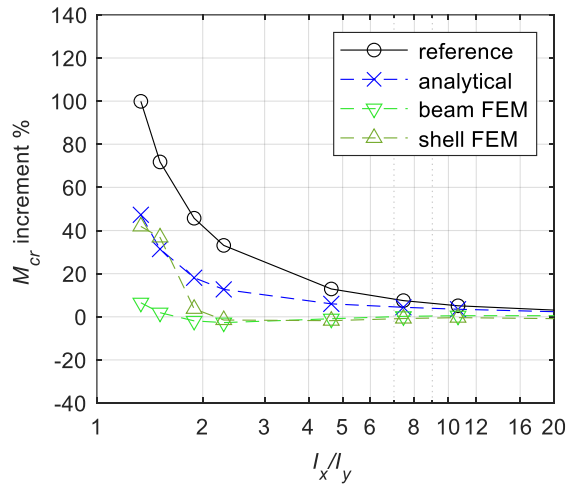
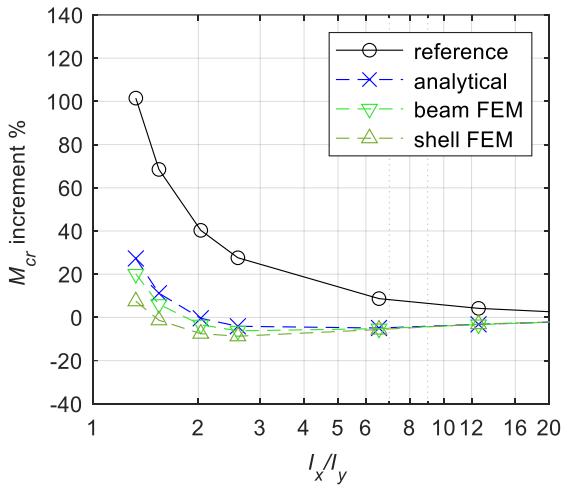


Figure 4: M_{cr} increments, FrFw-FrFw, DSI (left) and RHS (right)

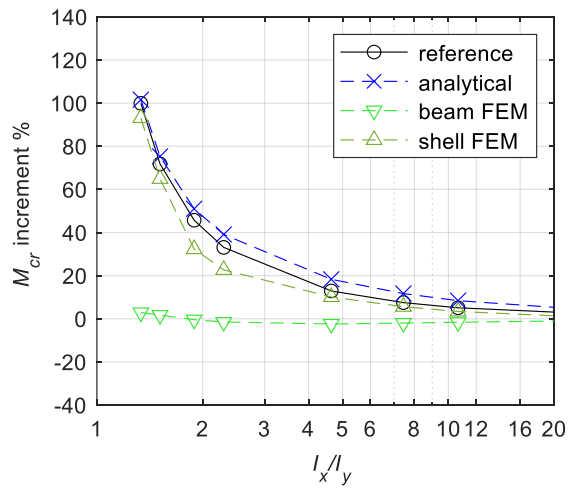
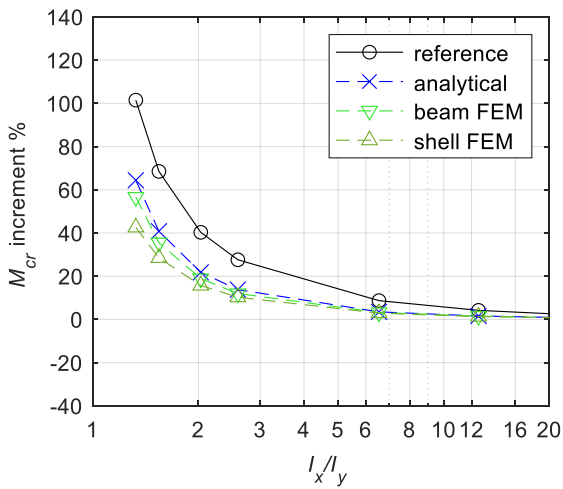


Figure 5: M_{cr} increments, PrPw-FrFw, DSI (left) and RHS (right)

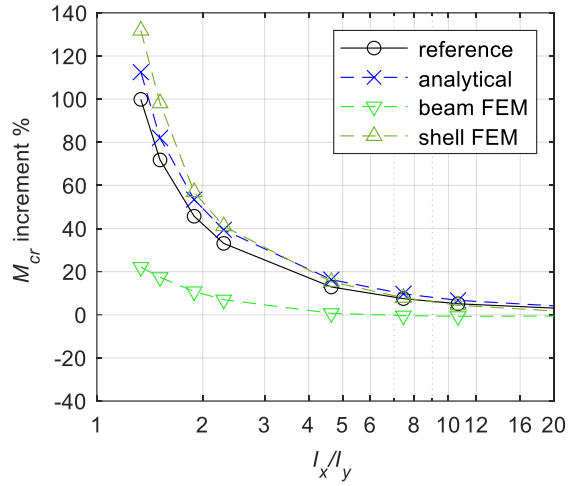
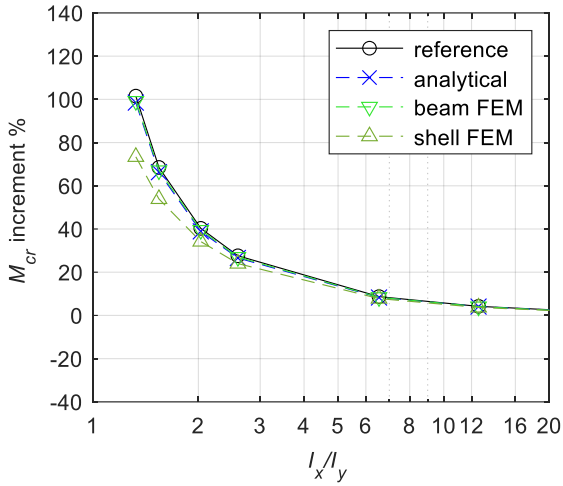


Figure 6: M_{cr} increments, PrFw-PrFw, DSI (left) and RHS (right)

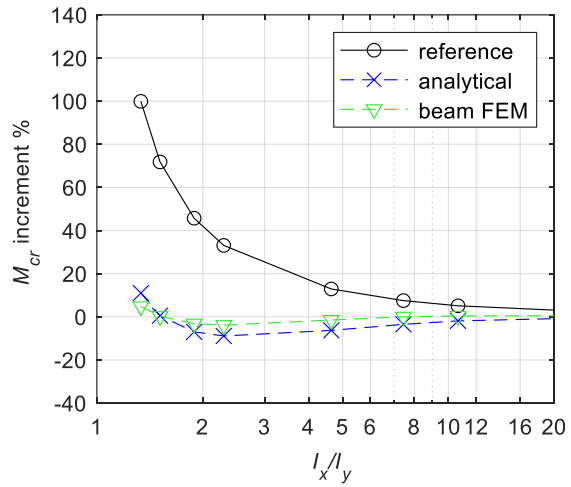
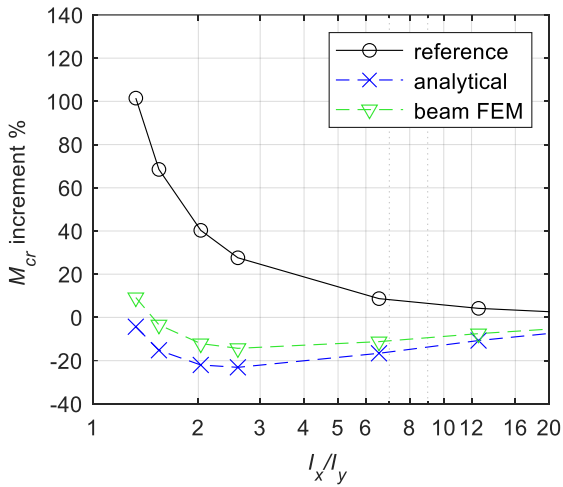


Figure 7: M_{cr} increments, FrPw-FrPw, DSI (left) and RHS (right)

5. Conclusions

It is known from the literature that the prebuckling deflection of a beam has an influence on the critical moment to lateral-torsional buckling. However, in previous investigations simply-supported beams were considered. In this paper analytical and numerical studies were presented to investigate how the prebuckling deflection influences the critical moment to lateral-torsional buckling of beams with various end support conditions. The main conclusions are as follows.

- Our results confirm the observation from the literature that the effect of prebuckling deflection becomes negligible if the ratio of the primary and lateral flexural stiffness of the beam (or: the ratio of the major-axis and minor-axis moment of inertia of the cross-section) is large.
- Approximate analytical solutions can be derived for various end support conditions.
- The end supports have important influence on the behavior. Unlike suggested by the previous literature, the effect of prebuckling deflection is not always positive. For example,

if the beam ends are fixed against lateral rotation, the prebuckling deflection decreases the critical moment in most cross sections.

- The results of numerical studies show that the value of the critical moment with considering the prebuckling deflection is somewhat dependent on the method used for the analysis.
- The analytical solution and beam finite element solution are nearly identical for I-shaped cross-section. However, significant discrepancy is observed in the case of rectangular hollow sections.
- The tendencies shown by the shell model solutions are always similar to those obtained by the analytical solutions, though small differences between the numerical results is experienced.

Acknowledgments

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